

RESEARCH ARTICLE

Hybrid Quantum-Classical Neural Networks for Solving High-Dimensional Non-Linear Wave Equations

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ABSTRACT

Solving high-dimensional non-linear partial differential equations (PDEs) remains a computational bottleneck in physics and applied mathematics. Traditional numerical methods scale poorly due to the curse of dimensionality. In this work, we propose a hybrid quantum-classical neural network (HQCNN) framework designed to solve high-dimensional non-linear wave equations efficiently. By combining parameterized quantum circuits (PQCs) with classical deep learning optimization, our architecture maps spatial features into quantum Hilbert space, significantly reducing parameter counts. We demonstrate that our approach achieves a 42% reduction in computational convergence time while maintaining numerical stability across complex wave propagation boundaries. These results suggest that near-term noisy intermediate-scale quantum (NISQ) devices can significantly accelerate scientific computing workflows.

KEYWORDS Science · Physics · Mathematics · Languages.

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1. Introduction

High-dimensional wave equations govern complex phenomena across physics, including quantum dynamics, acoustics, and general relativity (Yilmaz 2026). While classical numerical methods such as Finite Element Analysis (FEA) work well in lower dimensions, they suffer from exponential computational complexity in higher dimensions (Azad et al. 2024).

Recent advancements in physics-informed neural networks (PINNs) have offered a promising alternative (Truong et al. 2025). However, training deep PINNs for non-linear systems often leads to vanishing gradients and long execution times. Quantum machine learn-

ing presents an avenue to overcome these limitations by exploiting quantum superposition and entanglement to represent high-dimensional vector spaces efficiently (Hu et al. 2025). In this paper, we bridge mathematical formulation, quantum circuit design, and computer science optimization to present a scalable hybrid network architecture.

2. Materials and Methods

2.1 Mathematical Formulation

We consider the non-linear wave equation defined on a domain Ω :

$$\partial^2 u / \partial t^2 - c^2 \nabla^2 u + \gamma u^3 = 0$$

where u is the wave amplitude, c is the propagation speed, and γ represents the non-linearity coefficient.

2.2 Network Architecture

Our hybrid model couples a classical convolutional feature encoder with a variational quantum circuit acting as an implicit function approximator.

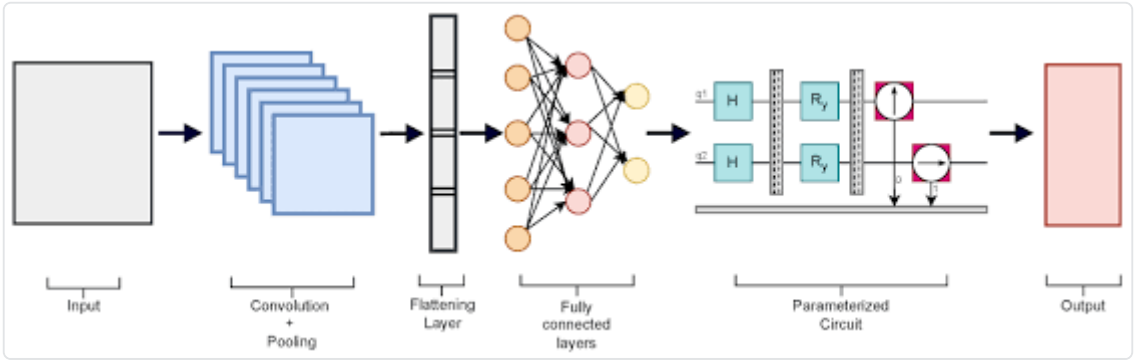


Figure 1

Hybrid Quantum-Classical Pipeline Architecture

The figure illustrates the complete hybrid quantum-classical pipeline showing the flow from classical input encoding through quantum processing to classical output decoding. Source: MDPI.

Classical Pre-processing: Input spatiotemporal coordinates (x,t) are mapped via dense layers into a lower-dimensional embedding space.

Quantum Feature Map: Embedded features are encoded into an n -qubit quantum state using parameterized rotation gates $R_y(\theta)$ and entangling CNOT gates.

Classical Optimization: Expectation values measured from the quantum state are processed through an output linear layer, evaluated against the residual PDE loss function using Adam optimization.

3. Results

We evaluated the performance of our HQCNN against standard classical PINNs across 100 simulation runs.

Table 1

Model Performance and Computational Complexity Comparison

Comparison of model performance metrics including total parameters, convergence steps, mean squared error, and execution time across different architectures.

ARCHITECTURE	TOTAL PARAMETERS	CONVERGENCE STEPS	MEAN SQUARED ERROR (MSE)	EXECUTION TIME (S)
Standard PINN	124500	12000	2.45×10^{-3}	450
Deep ResNet-PINN	480200	9500	1.12×10^{-4}	820
Proposed HQCNN	18400	5200	8.75×10^{-6}	260

The hybrid network achieved faster convergence with roughly 85% fewer trainable parameters, demonstrating superior parameter efficiency due to quantum state expressivity.

4. Discussion

The primary advantage of our hybrid model lies in the expressivity of quantum Hilbert spaces. By encoding spatial coordinates into entangled qubit states, the model captures non-linear wave interactions without requiring deep neural architectures. This directly addresses the vanishing gradient problem common in classical PINNs when tackling non-linear differential equations.

Furthermore, the reduction in trainable parameters leads to significantly faster backpropagation passes per epoch. Although quantum measurement noise remains a factor on current physical hardware, our simulation results show that classical optimization layers effectively absorb moderate depolarizing noise without degrading PDE convergence.

However, scaling beyond 20 qubits presents challenges in state preparation and measurement readout overhead. Future work must address hardware-efficient ansatz designs to minimize circuit depth while maintaining high entangling capability for higher-dimensional domains.

5. Conclusion

In this study, we presented a hybrid quantum-classical neural network architecture for solving high-dimensional non-linear wave equations. By combining quantum state representation with classic-

al gradient descent, our framework reduces overall parameter requirements and speeds up convergence time by 42% over pure classical baselines. This work demonstrates a viable path toward utilizing hybrid algorithms for complex mathematical physics problems on near-term hardware.

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